The probability that the 2nd song repeats the first is 1/100. If it was a repeat, we stop listening. With probability 99/100 it was not a repeat, so we go to the 3rd song. This repeats either the first or 2nd songs with probability 2/100. With probability 98/100 the 3rd song is not a repeat and we go to the 4th song. This repeats one of the first three songs with probability 3/100, etc.

So given that we get to song number $x \geq 2$, the probability that it is a repeat of an earlier song is

$$\frac{x-1}{100}.$$

From above, the probability that we get to song #2 is 1. The probability we get to song #3 is 99/100. The probability we get to song #4 is $98/100 \times 99/100$, etc. So the probability we get to song number $x \geq 2$ can be written as:

$$\frac{99!}{100^{x-2}\,(99-x+2)!}.$$

Putting these two things together, the probability that the first repeat occurs at song number $x \geq 2$ is:

$$\Pr\left(\text{First repeat at song } x\right) = \frac{99!}{100^{x-2}\,(99-x+2)!} \times \frac{x-1}{100}.$$

Now we want to find the mean of x. The possible values for x are between 2 and 100. Thus the mean value of the song number at which we hear the first repeat is:

$$\begin{aligned}
\mu &= \sum_{x=2}^{100} x \Pr\left(\text{First repeat at song } x\right) \\
&= \sum_{x=2}^{100} \left(\frac{99!}{100^{x-2}\,(99-x+2)!} \times \frac{x-1}{100} \times x \right)
\end{aligned}$$

Evaluating we get $\mu = 13.21$.